Geometry

Triangles :→ ① Scalene Triangle ② Isosceles Triangle ③ Equivalateral Δ

on basis of angles.

(a) Acute Δ → $a<b<c$ sides then. $\boxed{a^2+b^2>c^2}$

(b) Right Δ → $c>a, c>b$ then $a^2+b^2=c^2$

(c) obtuse Δ → $c>a, c>b$ then $a^2+b^2<c^2$

⇒ Conditions for formation of triangle sum of two side is greater than third

- $a+b>c$. • $a+c>b$ • $b+c>a$.

and $|a-b|<c$ $|b-c|<a$ $|c-a|<b$

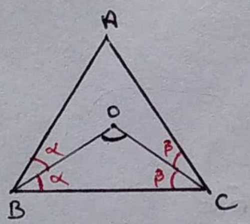
For ΔABC.

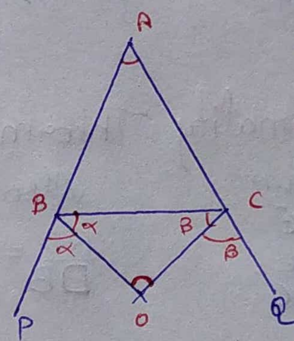
① $|b-c|<a<|b+c|$ ② $|a-c|<b<|a+c|$

③ $|a-b|<c<|a+b|$

Properties of a triangle

- If the angle bisectors of $\angle ABC$ and $\angle ACB$ of a triangle ABC meet at a point O then $\boxed{\angle BOC = 90^\circ + \frac{\angle A}{2}}$



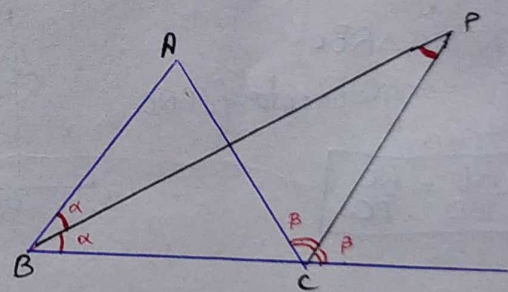


Bisector of $\angle PBC$ and $\angle QCB$ intersect at O then

$\boxed{\angle BOC = 90^\circ - \frac{\angle A}{2}}$

- The angle between internal bisector of one base angle and external bisector of other base angle of a triangle is equal to one half of vertical angle.

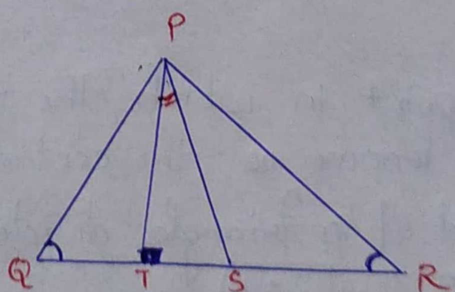
i.e → $\boxed{\angle BPC = \frac{1}{2}\angle BAC}$



- In ΔPQR.

PS is the bisector of $\angle QPR$ and $PT \perp QR$

Then $\boxed{\angle TPS = \frac{|\angle Q - \angle R|}{2}}$



↔ **Congruence of triangle** $\Delta ABC \cong \Delta DEF$

then $AB=DE$, $BC=EF$, $AC=DF$

$\angle A=\angle D$, $\angle B=\angle E$, $\angle C=\angle F$

Condition → Side angle side (SAS), ASA, AAS, SSS, RHS

Similarity of a triangle

:- Two triangles are similar if corresponding angle have same measure [OR] If the length of corresponding sides are proportional

$\triangle ABC \sim \triangle PQR$ then $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$.

Also if $\triangle ABC \sim \triangle PQR$.

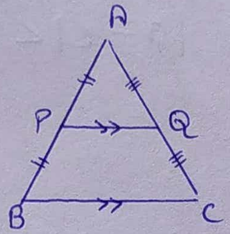
$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} = \frac{h_1}{h_2} = \frac{M_1}{M_2} = \frac{R_1}{R_2} = \frac{r_1}{r_2} = \frac{P_1}{P_2} = \sqrt{\frac{\Delta_1}{\Delta_2}}$$

Where. $h \rightarrow$ height, $M \rightarrow$ Median, $r \rightarrow$ inradius, $R \rightarrow$ circumradius

$P \rightarrow$ perimeter, $\Delta \rightarrow$ area

Mid point Theorm.

If $PQ \parallel BC$ &
P is midpoint of AB
and Q is midpoint of AC



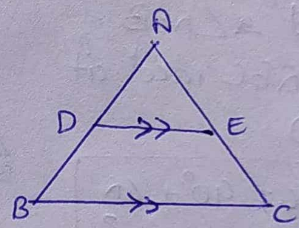
$\boxed{PQ = \frac{1}{2} BC}$

$$\frac{\text{Area } \triangle PQA}{\text{Area } \triangle BCA} = \frac{1}{4}$$

Basic Proportionality Theorm.

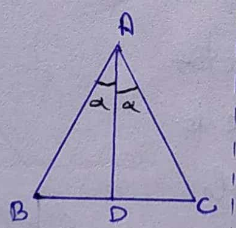
If $DE \parallel BC$ then
$\triangle ABC \sim \triangle ADE$
$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

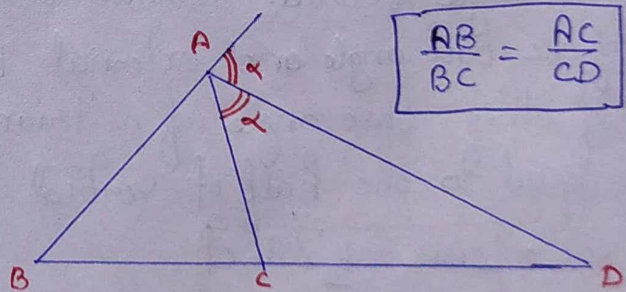
Also
$\frac{AD}{DB} = \frac{AE}{E}$



Angle bisector theorm.

(i) In $\triangle ABC$.
AD $\rightarrow$ angle bisector of $\angle BAC$
then
$\boxed{\frac{AB}{BD} = \frac{AC}{DC}}$





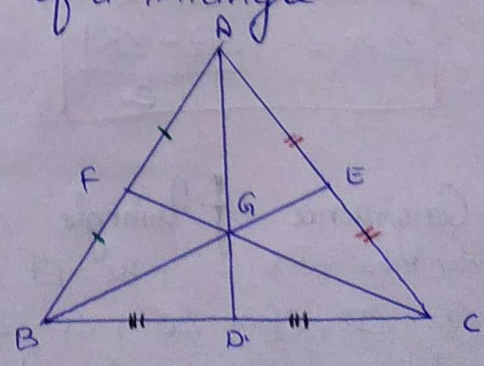
$\boxed{\frac{AB}{BC} = \frac{AC}{CD}}$

Centroid

It is the point in which the three medians of the triangle intersect is known as the centroid of a triangle

The centroid of a triangle divide the median in $\boxed{2:1}$

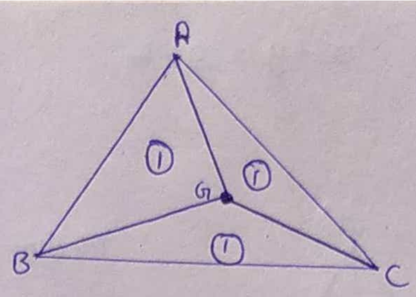
$G \rightarrow$ centroid
$AG : GD = 2:1$
$BG : GE = 2:1$
$CG : FG = 2:1$



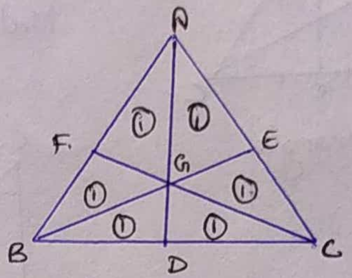
Area $\triangle ABD$ = Area $\triangle ADC$
Area $\triangle BCF$ = Area $\triangle ACF$
Area $\triangle ABE$ = Area $\triangle CBE$

$$AD^2 + BE^2 + CF^3 = \frac{3}{4}\left[AB^2 + BC^2 + AC^2\right]$$

- With respect to centroid triangle is divided into three parts of equal areas. The centroid divides the Δ in three equal parts.
  Area $AGB = AGC = BGC$



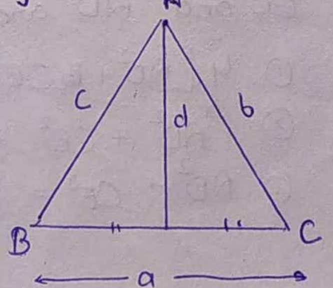
- All three median divide the triangle into 6 equal parts.



- $4(AD+BE+CF) > 3(AB+BC+CA)$.
- $4(AD^2+BE^2+CF^2) = 3(AB^2+BC^2+CA^2)$
- 4 (Area of triangle formed by median) $= 3$ (area of ΔABC).

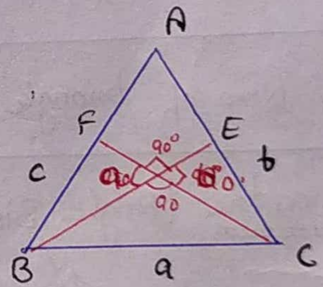
Length of medium

$$\boxed{AD = \frac{1}{2}\sqrt{2(AB+AC)^2 - BC^2}}$$



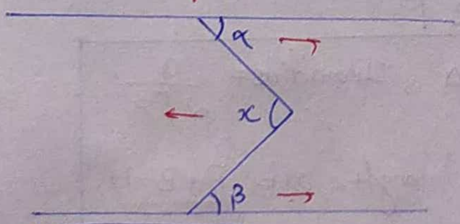
- If medians intersect at $90°$

$$5BC^2 = AB^2 + AC^2$$

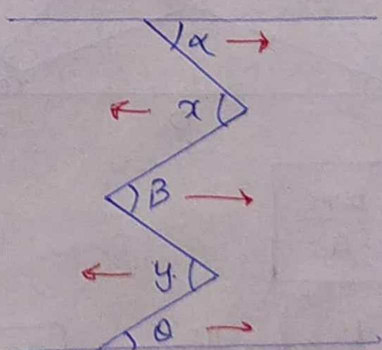


- ~~Relation of medians in a right angle~~

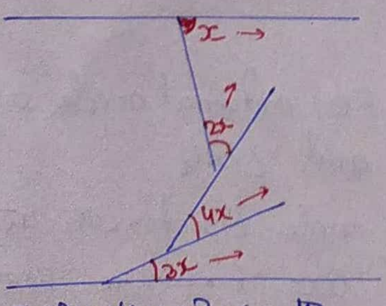
Arrow Concept



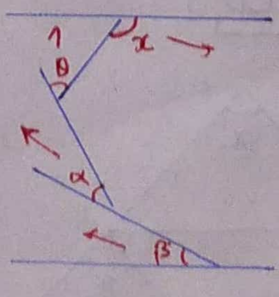
$$\boxed{x = \alpha + \beta}$$



$$\boxed{\alpha + \beta + \theta = x + y}$$

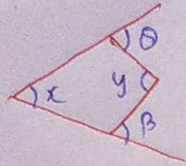


$x + 2x + 4x + 3x = 18$



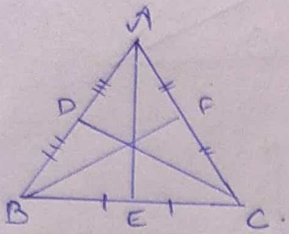
$x = \theta + \alpha + \beta$

4



$x + y = \theta + \beta$

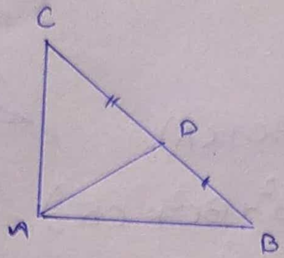
Median :



Median → opposite side को Two equal parts में Break कर देता है

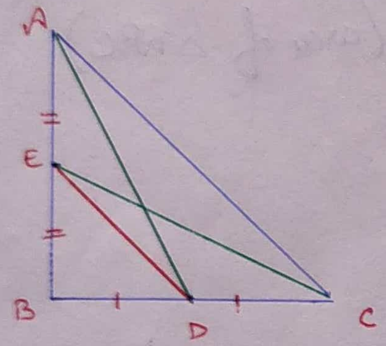
Median का Intersection point Centroid होता है

in Right angle triangle



If AD → median
then CD = DB
also

$\boxed{AD = CD = BD}$



EC and AD are medians.

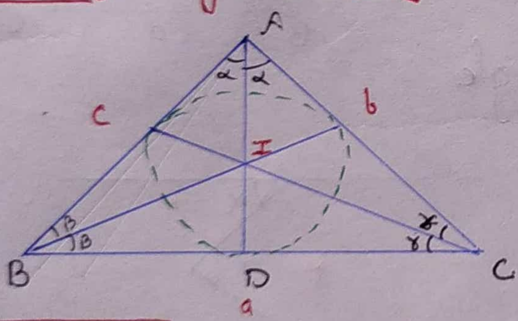
① $4[AD^2 + \cancel{B}CE^2] = 5AC^2$

② $AD^2 + CE^2 = 5ED^2$

③ $AD^2 + CE^2 = AC^2 + ED^2$

$\boxed{\text{Area of triangle} = \frac{4}{3}\,[\text{Area formed by its median}].}$

Incentre [Angle Bisector]



$r = \dfrac{\text{Area}}{\text{Semi radius}}$

$\boxed{\angle BIC = 90 + \frac{A}{2}}$

For Equilateral Δ = Inradius = $\dfrac{a}{2\sqrt{3}}$

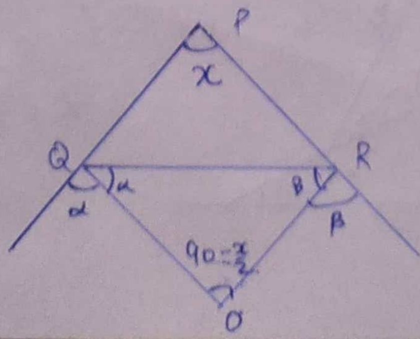
For Right angle triangle $r = \dfrac{P+B-H}{2}$

$r = \dfrac{R}{2}$

$\boxed{\dfrac{AI}{ID} = \dfrac{b+c}{a}}$

Angle bisector theorm $\boxed{\dfrac{AB}{BD} = \dfrac{AC}{CD}}$

External angle bisector.



QO and RO external angle bisector of ∠PQR and ∠PRQ

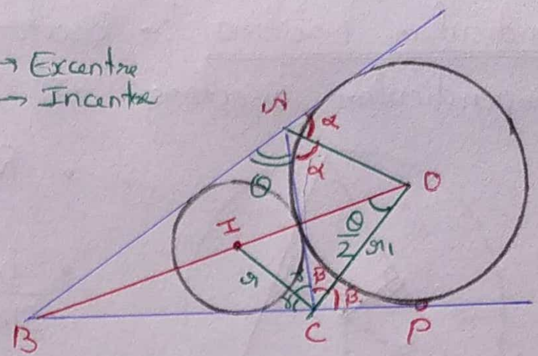
External angle bisector जहाँ पर मिलते हैं वो Excentre होता है

Point 'O' is also the excentre of △ABC

$$\boxed{\angle BOC = \frac{1}{2}\angle BAC}$$

$$\boxed{\beta+\gamma = \frac{\pi}{2}}$$

O → Excentre
I → Incentre

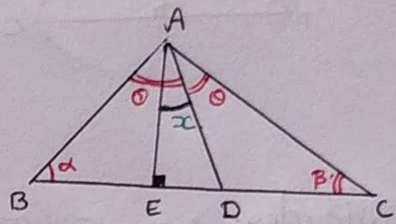


$$\boxed{r^2 + r_1^2 = (IO)^2}$$

:- r and r_1 are not radius

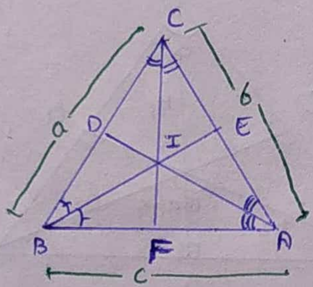
⇄ In △ABC, AD is bisector of ∠BAC & AE ⊥ BC
then

$$\boxed{\angle EAD = \frac{\angle ABC - \angle ACB}{2} \qquad x = \frac{\alpha-\beta}{2}}$$



$$\boxed{\angle BAE = \theta - x}$$

:⇄

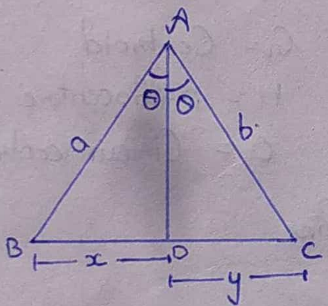


If AD, BE & CF are angle bisectors of ∠A, ∠B & ∠C respectively the

$$\frac{CI}{IF} = \frac{a+b}{c}, \quad \frac{BI}{IE} = \frac{a+c}{b}, \quad \frac{AI}{ID} = \frac{b+c}{a}$$

:- Also $\cancel{IE}$ $\frac{IF}{CF} + \frac{IE}{BE} + \frac{ID}{AD} = 1$ and $\frac{CF}{IF} + \frac{BE}{IE} + \frac{AD}{ID} = 2$

:→

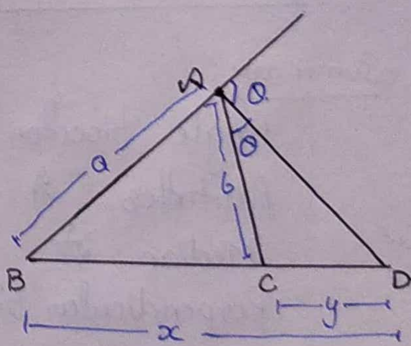


AB = a, AC = b, BD = x, CD = y.
and AD is angle bisector of A then

$$\boxed{AD^2 = ab - xy}$$

:→

$$\boxed{AD^2 = xy - ab}$$



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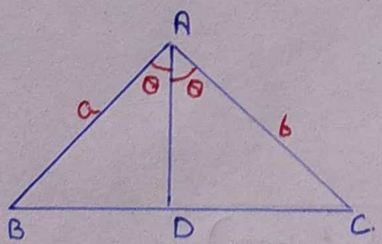
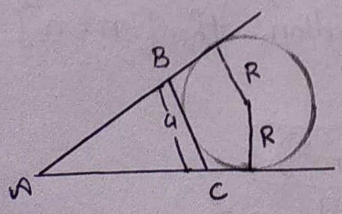


$$\boxed{AD = \frac{2ab\cos\theta}{(a+b)}}$$

:→ Excentre



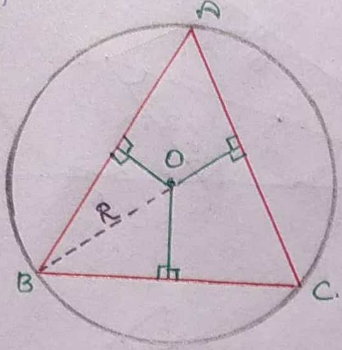
$$R = \frac{\Delta}{s-a}$$

Δ → Area of triangle
S → semiperimeter.
R → Exradius

⑥

Perpendicular bisectors :- Bisectors which are perpendicular

Perpendicular bisectors जहाँ पर मिलते हैं उसे Circumcentre कहते हैं



- $R = \frac{abc}{4\Delta}$ where Δ = area.
- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R.$

Equivilater Δ Circumradius $= \frac{a}{\sqrt{3}}$

Right angle Δ Circumradius $= \frac{\text{Hypotenuse}}{2}$

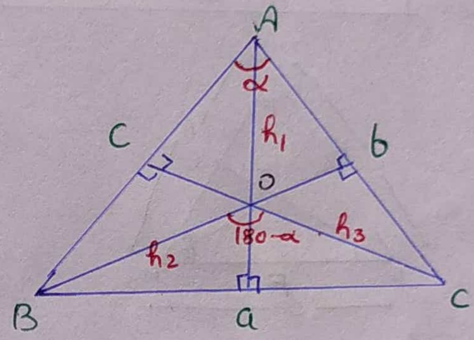
Altitudes :- जहा पर Perpendicular मिलते हैं। जरूरी नही की Perpendicular bisector हो।

इसमें Orthocentre बनता है

$$\boxed{\angle BOC = 180 - \angle A}$$

$h_1 : h_2 : h_3 = \frac{1}{a} : \frac{1}{b} : \frac{1}{c}.$

$$\boxed{\frac{1}{r} = \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3}}$$



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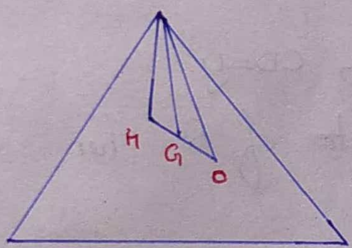


$\boxed{HG : GO = 2:1}$

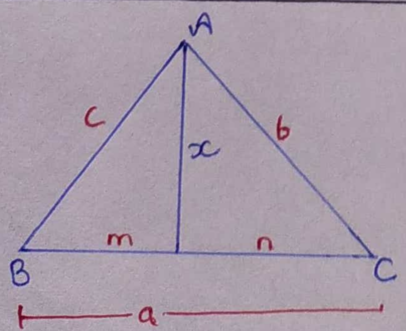
Here G = Centroid
H = orthocentre
O = Circumcentre

G, H, O lies on same line.

Summary

Angle bisector से → Incentre
Altitudes से → orthocentre
Median से → Centroid.
Perpendicular bisector से → Circumcentre

⑥ Stewart theorm and its extension.



$$\boxed{c^2n + b^2m = a(x^2 + mn)}$$

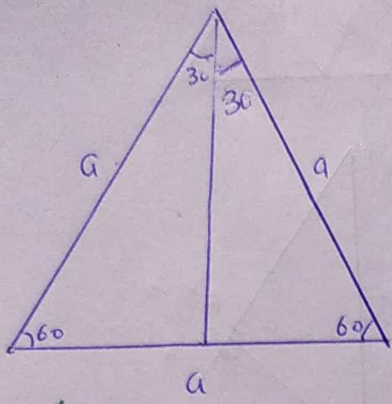
Case 1 - If AD is median then $[m = n]$
Appollonius theorm.

$$\boxed{c^2 + b^2 = 2(x^2 + m^2).}$$

Case → 2 Isoscles. $[c = b]$.

$$\boxed{c^2 = x^2 + mn}$$

Equilateral triangle:-



Area = $\frac{\sqrt{3}}{4}a^2$, Height = $\frac{\sqrt{3}}{2}a$

orthocentre, Circumcentre, Incentre, Centroid → all are on same point..

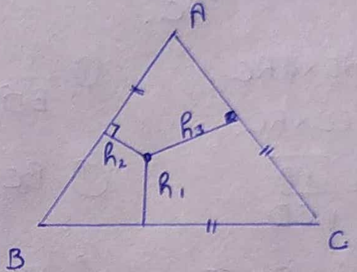
Inradius :→ $\frac{a}{2\sqrt{3}}$

Circumradius :→ $\frac{a}{\sqrt{3}}$

Generalisations

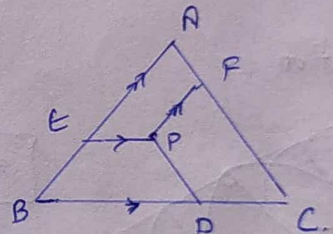
1. If 'p' is any point inside the triangle and H is height of equilateral triangle then.

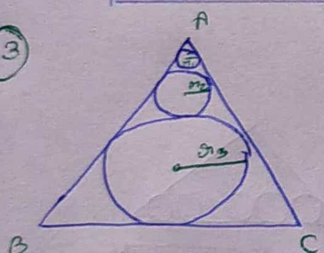
$$\boxed{H = h_1 + h_2 + h_3}$$



2. Suppose P is a point inside equilateral triangle such that PE∥BC, PF∥AB & PD∥AC then.

$$\boxed{PE + PF + PD = AB = AC = BC}$$



3. 

$r_1 : r_2 : r_3 = 1 : 3 : 9$.

radius in Geometric progression ~~for~~

For n circles = 1 : 3 : 9 : 27 : . . . . .

→ Relation between Inradius, Circumradius and Exradius

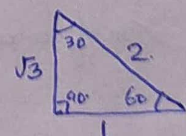
ΔABC → Equilateral Δ'

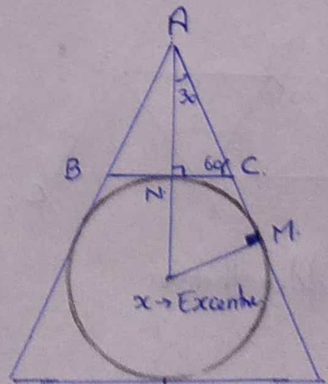
If $XM = x$

then $AX = 2x$

$NX = XM = x$

[Ex radius].





⇒ $AN = x$

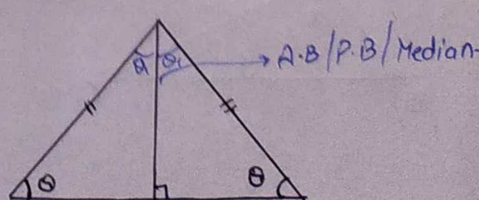
AN = height of equilateral Δ.

In radius = $\frac{x}{2}$, Circumradius = $\frac{2x}{3}$, Exradius = 2

$$\boxed{\text{Inradius : Circumradius : Ex-radius} = 1 : 2 : 3}$$

Isosceles Triangle.

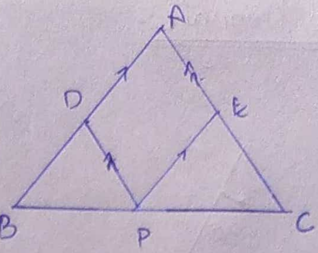
Incenter : Circumcentre : orthocentre : Centroid → Collinear



(8) Generalisations

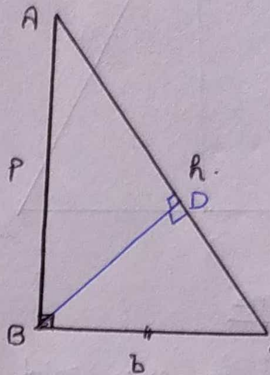
(1) If $\triangle ABC$ is isosceles ($AB = AC$) and P is any point on BC such that $DP \parallel AC$ & $EP \parallel AB$ then

$$\boxed{DP + EP = AB = AC}$$



(6) Right angle triangle

p → Perpendicular.
b → base length
h → hypotenuse



Tools

(1) $BD = \dfrac{AB \times BC}{AC}$.

(2) $AB^2 = AD \times AC$.

(3) $BC^2 = CD \times AC$.

(4) $\dfrac{AB}{BC} = \sqrt{\dfrac{AD}{CD}}$

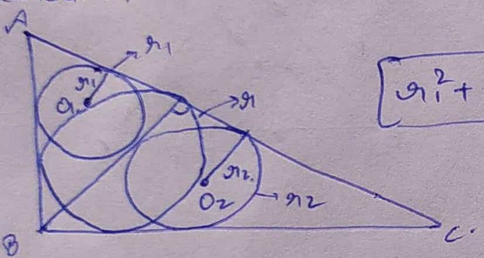
(5) $BD^2 = AD \times CD$

(6) $\dfrac{1}{BD^2} = \dfrac{1}{AB^2} + \dfrac{1}{BC^2}$.

(7) $BC = \dfrac{AB \times BD}{AD}$

(8) $AB = \dfrac{BC \times BD}{CD}$

Generalisation

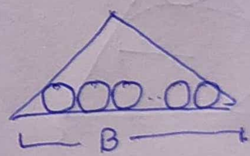


$\boxed{r_1^2 + r_2^2 = r^2}$, $\boxed{O_1O_2 = \sqrt{2}\,r}$ → Distance b/w two circle

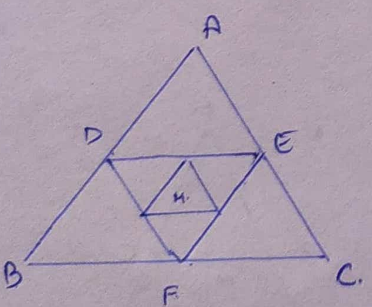
(c) ~~Similarity~~ Some important Results.

(1) If there are 'n' Circle on base of any triangle

$$r = \frac{BR}{2R(n-1)+B}$$



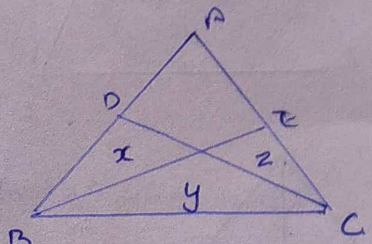
(2)



sum of area of triangle $= \dfrac{4}{3}$ (Area of bigger $\triangle$).

Sum Perimeter of all triangle = 2 (Perimeter of bigger $\triangle$)

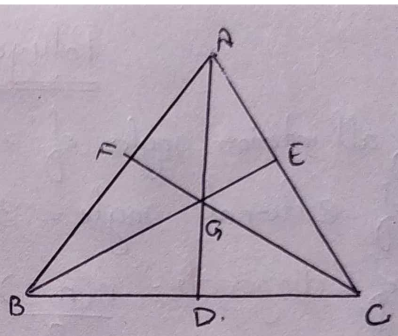
(3) Ladder theorem



$$\frac{1}{\Delta} + \frac{1}{y} = \frac{1}{x+y} + \frac{1}{y+z}$$

Cevians

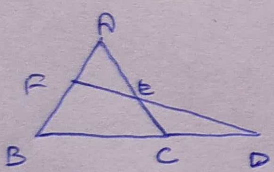
① $\frac{AF}{BF} \times \frac{BD}{CD} \times \frac{CE}{EA} = 1.$



② $\frac{GD}{AD} + \frac{GE}{BE} + \frac{GF}{CF} = 1$

③ $\frac{AD}{GD} + \frac{BE}{GE} + \frac{CF}{GF} = 2$

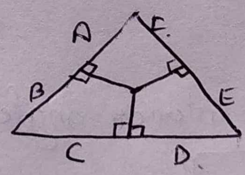
→



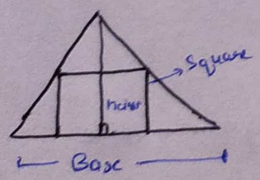
$\frac{BF}{FA} \times \frac{AE}{EC} \times \frac{CD}{BD} = 1$

Carnot's Theorm

$$\boxed{A^2 + C^2 + E^2 = B^2 + D^2 + F^2}$$

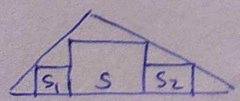


⊙



$= \frac{1}{\text{Side of square}} = \frac{1}{\text{Base}} + \frac{1}{\text{height}}$

⊙



$\sqrt{S} = \sqrt{S_1} + \sqrt{S_2}$

Polygon

1. Sum of all interior angles of a polygon of side 'n' $= (n-2) \times 180$

Sum of external angle $= 360$

- No. of diagonals $= \dfrac{n(n-3)}{2}$

→ Area of regular Polygon $= \dfrac{na^2}{4} \cot\left(\dfrac{\pi}{n}\right)$

Inradius $= \dfrac{a}{2} \cot\left(\dfrac{\pi}{n}\right)$

Circumradius $= \dfrac{a}{2} \operatorname{cosec}\left(\dfrac{\pi}{n}\right)$

- For $n \geq 5$

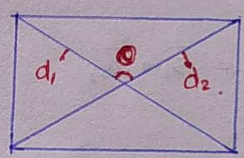
If Exterior angle $= x$, interior angle $= kx$ [k times]

then $\boxed{n = 2k+2}$

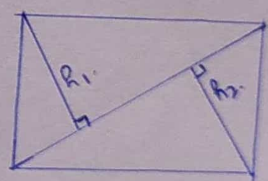
- **Quadrilaterals** :→ Have 4 sides → Rectangle, square, rhombus, llgram, kite

General formulas

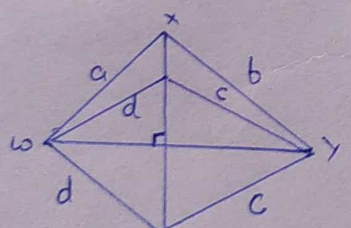
1. Area $= \dfrac{1}{2} \times d_1 \times d_2 \times \sin\theta$



2. Area $= \dfrac{1}{2} \times BD \times (h_1 + h_2)$



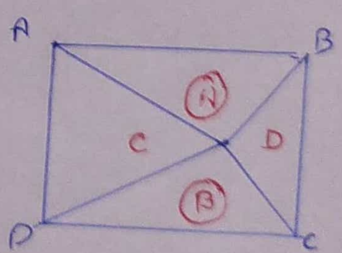
3. $\boxed{a^2 + c^2 = b^2 + d^2}$



4. A, B, C and D are areas.

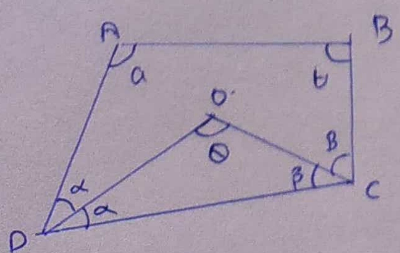
then $\boxed{A+B = C+D}$

$\boxed{A \times B = C \times D}$

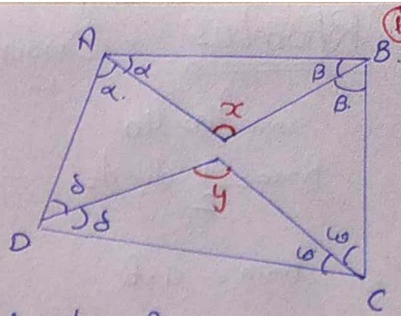


5. If DO & CO are angle bisectors then

$\theta = \dfrac{a+b}{2}$



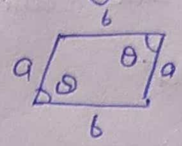
6 $\boxed{x+y = \pi = 180^\circ}$



→ **Parallelogram**

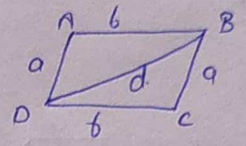
opposite lines parallel, opposite angle equal, digonals bisect each other

→ Area $= \frac{1}{2} \times a \times b \times \sin(\theta)$.

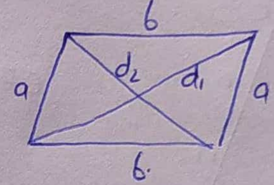


→ Area ABCD $= 2\sqrt{s(s-a)(s-b)(s-d)}$

$s = \frac{a+b+d}{2}$

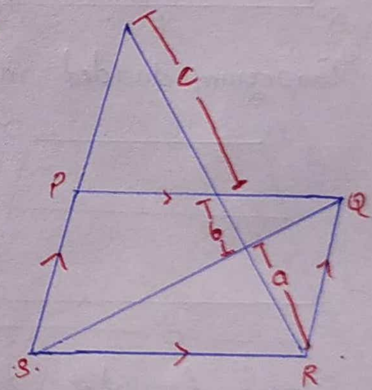


- Bisectors of angles of parallelogram form a rectangle
- If ABCD is llgram then $2(a^2+b^2) = d_1^2 + d_2^2$



→ Parallelogram that circumscribes a circle is rhombus

- If PQRS is a parallelogram then $\boxed{a^2 = b(b+c)}$

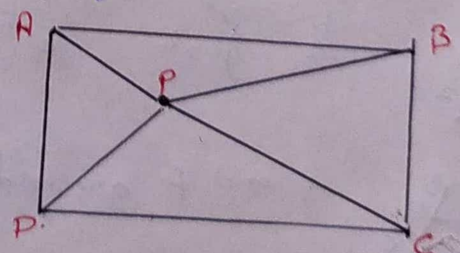


→ **Rectangle**

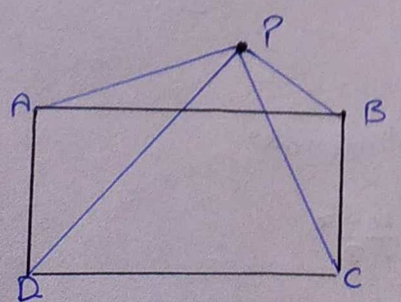
1. For equal Perimeter, area of rectangle is always greater than area of parallelogram.
2. If one side & Area of rectangle and area of parallelogram are equal then Perimeter of llgram > Perimeter of Rectangle

British Flag theorm.

1. $\boxed{AP^2 + PC^2 = PB^2 + PD^2}$



P → point inside

2. 

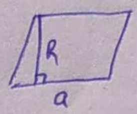
P is any point outside of rectangle

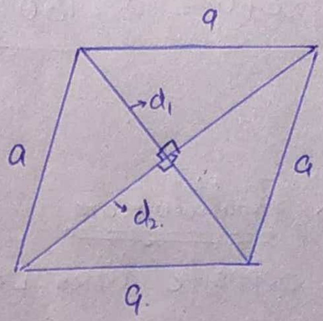
$\boxed{PA^2 + PC^2 = PB^2 + PD^2}$

12. **Rhombus** : Diagonal are perpendicular

Perimeter $= 4a$.

Area $= \frac{1}{2} \times d_1 \times d_2 = \frac{d_1 d_2}{2}$





Area $= a \times h$

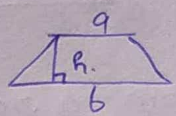
:-> Sum of square of side = Sum of square of diagonals

$= d_1^2 + d_2^2 = 4a^2$

Square :- all angle -> $90°$

• **Trapezium**

• Area $= \frac{1}{2}(a+b)h$



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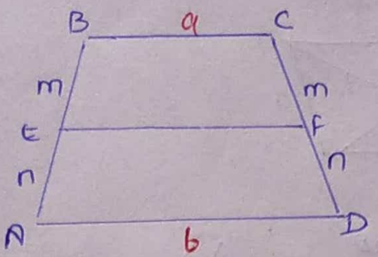
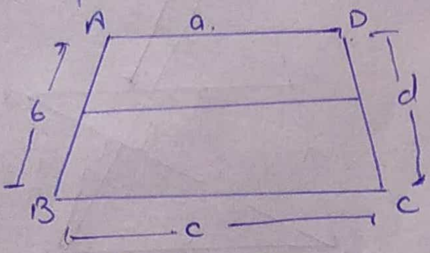


$EF = \frac{mb+na}{m+n}$

$\frac{BE}{EA} = \frac{CF}{FD} = \frac{m}{n}$

• Trapezium divided into 2-equal parameters.



If $\frac{AE}{EB} = \frac{DF}{FC} = x$

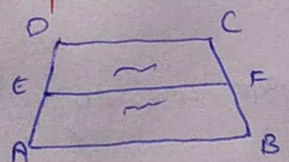
Find x

EF bisects into 2 equal parameter

Direct formula $x = \frac{S-a}{S-c}$

here $S = \frac{a+b+c+d}{2}$

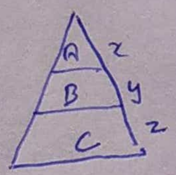
• Trapezium divided into equal area.



Area AEFB = Area EFCD

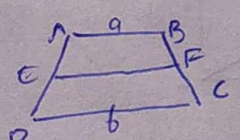
$EF = \sqrt{\frac{a^2+b^2}{2}}$

•



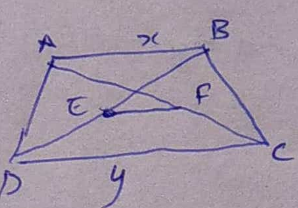
$\frac{A}{B} = \frac{x^2}{y^2 - x^2}$, $\frac{B}{C} = \frac{y^2 - x^2}{z^2 - y^2}$

•



E and F are mid points then $EF = \frac{a+b}{2}$.

•



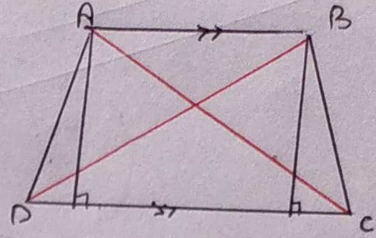
E and F are mid points of diagonals =

$EF = \frac{y-x}{2}$

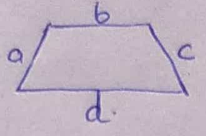
Diagonals of Trapezium

If ABCD is trapezium

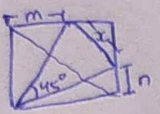
$$\boxed{AC^2 + BD^2 = AD^2 + BC^2 + 2(AB)(CD)}$$



Area of Trapezium

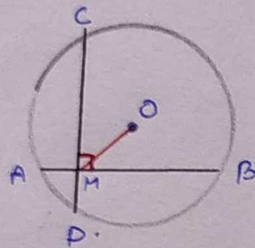


$S = \frac{(a+b+c+d)}{2}$, Area $= \frac{d+b}{d-b}\sqrt{(s-b)(s-d)(s-b-a)(s-b-c)}$



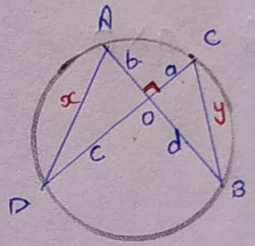
$\boxed{x = m+n}$ when square

Circle



If $CD \perp AB$, $AB = 2b$, $CD = 2a$ & $OM = c$.

then radius $= r = \sqrt{\frac{a^2+b^2+c^2}{2}}$



$AD = x$, $BC = y$

then $\boxed{x^2 + y^2 = 4R^2}$

or

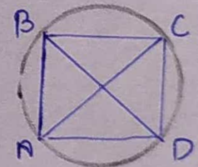
$a^2 + b^2 + c^2 + d^2 = 4R^2$

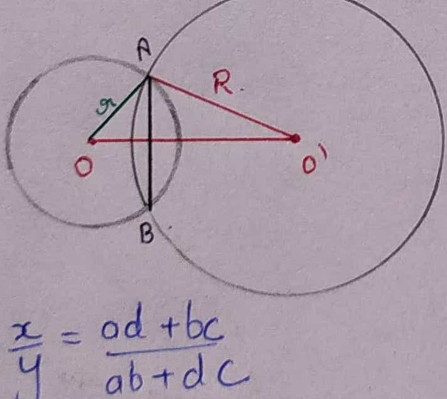
AO = b
CO = a
DO = c
BO = d

~~AC+BD~~

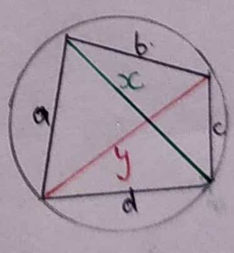
Ptolemy theorm

$AC \times BD = AB \times CD + BC \times AD$



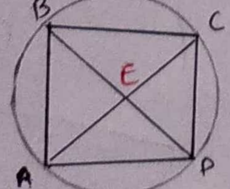


$AB = \frac{2Rr}{OO'}$



$\frac{x}{y} = \frac{ad+bc}{ab+dc}$

Imp

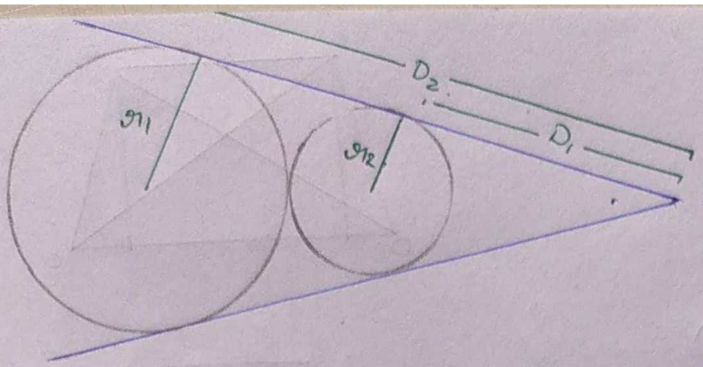


$\frac{AE}{EC} = \frac{AB \times AD}{BC \times CD}$

when $AE = EC$

$AB \times AD = BC \times CD$

(14)

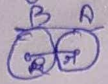


$$D_1 = \frac{2r_2\sqrt{r_1 r_2}}{r_1 - r_2}$$

$$D_2 = \frac{2r_1\sqrt{r_1 r_2}}{r_1 - r_2}$$

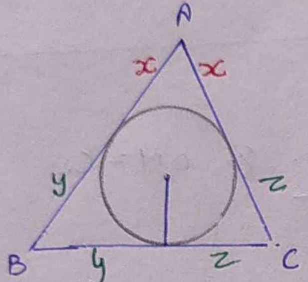
also be solved by similarity

- Transverse Common tangent = $\sqrt{d^2 - (r_1+r_2)^2}$
- Direct Common tangent = $\sqrt{d^2 - (r_1-r_2)^2}$
- When ~~$r_1 = r_2$~~ touch each other ⇒



$AB = 2\sqrt{Rr}$

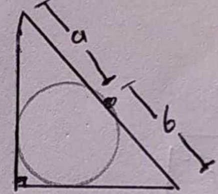
-



$$R = \sqrt{\frac{xyz}{x+y+z}}$$

$$\text{Area} = \sqrt{xyz(x+y+z)}$$

-



Area of Triangle = $a \times b$